5.2 Verifying Trigonometric Identities

There is no well-defined set of rules to follow in verifying trigonometric identities, and it is best to learn the process by **practicing.**

In general here are some guidelines for verifying trigonometric identities:

Guidelines for Verifying Trigonometric Identities

1. Work with one side of the equation ~~at a time~~. It is often better to work with the more complicated side first.

2. Look for opportunities to factor an expression, add fractions, square a binomial, or create a monomial denominator.

3. Look for opportunities to use the fundamental identities. Note which functions are in the final expression you want. Sines and cosines pair up well, as do secants and tangents, and cosecants and cotangents.

4. If the preceding guidelines do not help, then try converting all terms to sines and cosines.

5. Always try *something*. Even making an attempt that leads to a dead end can provide insight.

When verifying an identity, you cannot *assume* that the two side s of the equation are equal because you are trying to verify that they *are* equal.

In addition, you cannot use operations such as adding the same quantity to each side of the equation or cross multiplication.

Can only change <u>one</u> <u>side</u> to match the other!!!!

Ex: 1 Verify $\dfrac{\sec^2\theta - 1}{\sec^2\theta} = \sin^2\theta$

$\dfrac{\tan^2\theta}{\sec^2\theta}$

$\dfrac{\;\dfrac{\sin^2\theta}{\cos^2\theta}\;}{\;\dfrac{1}{\cos^2\theta}\;}$

$\dfrac{\sin^2\theta}{\cos^2\theta} \cdot \dfrac{\cos^2\theta}{1}$

$\sin^2\theta = \sin^2\theta$

You Try: Verify $\dfrac{\csc^2\theta - \cot^2\theta}{\tan^2\theta\csc^2\theta} = \cos^2\theta$

$$\frac{1}{\tan^2\theta\csc^2\theta}$$

$$\frac{1}{\frac{\cancel{\sin^2\theta}}{\cos^2\theta}\cdot\frac{1}{\cancel{\sin^2\theta}}}$$

$$\cos^2\theta = \cos^2\theta$$

Ex: 2 Verify $2\sec^2\alpha = \dfrac{1}{1-\sin\alpha} + \dfrac{1}{1+\sin\alpha}$

$$\frac{1+\cancel{\sin\alpha}+1-\cancel{\sin\alpha}}{(1-\sin\alpha)(1+\sin\alpha)}$$

$$\frac{2}{1-\sin^2\alpha}$$

$$\frac{2}{\cos^2\alpha}$$

$$2\sec^2\alpha = 2\sec^2\alpha$$

You Try: Verify $\dfrac{1}{\sec y - 1} + \dfrac{1}{\sec y + 1} = 2\cot y\csc y$

$$\frac{\sec y+\cancel{1}+\sec y-\cancel{1}}{(\sec y-1)(\sec y+1)}$$

$$\frac{2\sec y}{\sec^2 y - 1}$$

$$\frac{2\sec y}{\tan^2 y}$$

$$\frac{\frac{2}{\cos y}}{\frac{\sin^2 y}{\cos^2 y}}$$

$$\frac{2}{\cancel{\cos y}}\cdot\frac{\cos^{\cancel{2}} y}{\sin^2 y}$$

$$2\cdot\frac{\cos y}{\sin y}\cdot\frac{1}{\sin y}$$

$$2\cot y\csc y = 2\cdot\cot y\cdot\csc y$$

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:: 3 **Verify** $(\tan^2 x+1)(\cos^2 x-1)=-\tan^2 x$

$\sec^2 x \cdot (-1)(1-\cos^2 x)$

$-\sec^2 x \cdot \sin^2 x$

$-\frac{1}{\cos^2 x} \cdot \sin^2 x$

$-\frac{\sin^2 x}{\cos^2 x}$

$-\tan^2 x \quad = \quad -\tan^2 x$

You Try: **Verify** $(\csc^2 \theta-1)(\cos^2 \theta-1)=-\cos^2 \theta$

$(\cot^2\theta)(-1)(1-\cos^2\theta)$

$\left(\frac{\cos^2\theta}{\cancel{\sin^2\theta}}\right)(-1)\cancel{\sin^2\theta}$

$-\cos^2\theta \quad = \quad -\cos^2\theta$

Ex: 4 **Verify** $\tan x+\cot x=\sec x\csc x$

$\frac{\sin x}{\cos x}+\frac{\cos x}{\sin x}$

$\frac{\sin^2 x+\cos^2 x}{\cos x \cdot \sin x}$

$\frac{1}{\cos x \cdot \sin x}$

$\sec x \cdot \csc x \quad = \quad \sec x \csc x$

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ou Try: **Verify** $\sec x - \cos x = \sin x \tan x$

$$\frac{1}{\cos x} - \cos x$$

$$\frac{1-\cos^2 x}{\cos x}$$

$$\frac{\sin^2 x}{\cos x} = \frac{\sin x}{\cos x} \cdot \sin x \longrightarrow \sin x \cdot \tan x = \sin x \tan x$$

Ex: 5 **Verify** $\sec x + \tan x = \frac{\cos x}{(1-\sin x)} \frac{(1+\sin x)}{(1+\sin x)}$ ← do this to create a pythagorean identity

$$\frac{\cos x(1+\sin x)}{1-\sin^2 x}$$

$$\frac{\cancel{\cos x}(1+\sin x)}{\cos^{\cancel{2}} x}$$

$$\frac{1}{\cos x} + \frac{\sin x}{\cos x}$$

Seperate fractions to get what you want

$\sec x + \tan x =$ $\sec x + \tan x$

You Try: **Verify** $\csc\beta + 1 = \frac{\cot\beta}{(\sec B - \tan\beta)} \frac{(\sec\beta + \tan\beta)}{(\sec\beta + \tan\beta)}$

$$\frac{\cot\beta(\sec\beta + \tan\beta)}{\sec^2\beta - \tan^2\beta = 1}$$

$$\frac{\cos\beta}{\sin\beta} \cdot \frac{1}{\cos\beta} + \frac{\cancel{\cos\beta}}{\cancel{\sin\beta}} \cdot \frac{\cancel{\sin\beta}}{\cancel{\cos\beta}}$$

$$\frac{1}{\sin\beta} + 1$$

$$\csc\beta + 1$$

$\csc\beta + 1 =$

Ex: 6 **Verify** $\frac{\cot^2\theta}{1+\csc\theta} = \frac{1-\sin\theta}{\sin\theta}$

$$\frac{\csc^2\theta - 1}{1+\csc\theta}$$

$$\frac{(\csc\theta - 1)\cancel{(\csc\theta+1)}}{\cancel{1+\csc\theta}}$$

$$\csc\theta - 1$$

$$\frac{1}{\sin\theta} - 1$$

$$\frac{1-\sin\theta}{\sin\theta} = \frac{1-\sin\theta}{\sin\theta} \quad \checkmark$$

You Try: **Verify** $\frac{\sin\theta\tan^2\theta}{\sec\theta - 1} = \frac{\cos\theta+1}{\cot\theta}$

$$\frac{\sin\theta(\sec^2\theta - 1)}{\sec\theta - 1}$$

$$\frac{\sin\theta\cancel{(\sec\theta - 1)}(\sec\theta+1)}{\cancel{\sec\theta - 1}}$$

$$\sin\theta \cdot \left(\frac{1}{\cos\theta} + 1\right)$$

$$\sin\theta \cdot \frac{1+\cos\theta}{\cos\theta} = \tan\theta\,(1+\cos\theta)$$

$$\frac{\cos\theta + 1}{\cot\theta} = \frac{\cos\theta + 1}{\cot\theta} \quad \checkmark$$